Basic Operations with Complex Numbers:

Addition and subtraction of complex numbers works in a similar way as that of addition and subtraction of surds, Since the imaginary number i is defined as $i = \sqrt{(-1)}$.

Rules for Addition of Complex Numbers:

Add real parts, add imaginary parts.

Rules for subtraction of Complex Numbers:

Subtract real parts, subtract imaginary parts.

Examples:

1. $(6 + 7i) + (3 - 5i)$ = $(6 + 3) + (7 - 5)i = 9 + 2i$

2. $(12 + 6i) - (4 + 5i)$ = $(12 - 4) + (6 - 5)i = 8 + i$

Multiplication of Complex Numbers :

Expand brackets as usual, but care with i^2 .

Examples:

Q.1. $5(2 + 7i)$ $= 10 + 35i$

Q.2. $(6 - i) . (5i)$ = $30i - 5i^2 = - 5(-1) + 30i = \mathbf{5+30}i$

Q.3. $(2 - i) . (3 + i)$ = $6 - 3i + 2i - i^2 = 6 - (-1) - i = 6+1-i = 7 - i$

Q.4. $(5 + 3i)^2 = 25 + 2(5)(3)i + 9(i^2) = 25 + 30i + 9(-1) = 25-9+30i = 16 + 30i$

Q.5 $(2\sqrt{-9} - 3)(3\sqrt{-16} - 1)$

$= (2j(3) - 3)(3j(4) - 1)$

$= (6j - 3)(12j - 1)$

$= 72(j^2) - 36j - 6j + 3$

$= -69 - 42j$

Q.6 (3 + 2*i*)(3 – 2*i*) = $(3)^2 - (2i)^2 = 9 - 4i^2$ = 9 – (-4) = 9 + 4 = 13

Example:

3 + 2*i* is the **conjugate** of 3 – 2*i*. In general: $x + yi$ is the **conjugate** of $x - yi$ and $x - yi$ is the **conjugate** of $x + yi$.

Notice that when we multiply conjugates, our final answer is **Real** only it does not contain any imaginary terms.

The idea of **conjugate will be applied** when dividing complex numbers**.**

Division of Complex Numbers:

$$\frac{5}{3-\sqrt{2}}$$

We multiplied numerator and denominator by the **conjugate** of the denominator, 3 + √2:

$$\frac{5}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{15+5\sqrt{2}}{9-2} = \frac{15+5\sqrt{2}}{7}$$

Because there should be no radical (square root) in the denominator Dividing with complex numbers is similar**.**

Examples: Express in the form $x + yj$. $\frac{3-j}{4-2j}$

Solution:

The **conjugate** of 4 − 2*j* is **4 + 2*j*.**

$$\frac{3-j}{4-2j}\times\frac{4+2j}{4+2j}=\frac{12+6j-4j-2j^2}{16-4j^2}$$

$$=\frac{12+2+6j-4j}{16+4}$$

$$=\frac{14+2j}{20}$$

$$=\frac{7+j}{10}$$

2. Simplify: $\dfrac{1-\sqrt{-4}}{2+9j}$

Answer:

$$\frac{1-\sqrt{-4}}{2+9j}=\frac{1-2j}{2+9j}\times\frac{2-9j}{2-9j}$$

$$=\frac{2-9j-4j+18j^2}{4-81j^2}$$

$$=\frac{-16-13j}{4+81}$$

$$=\frac{-16-13j}{85}$$

Exercises:

1. Express in the form $a + bi$:

$$(4+\sqrt{-16})+(3-\sqrt{-81})$$

Answer:

$$(4+4j)+(3-9j) = 7-5j$$

2. Express in the form $a + bj$.

$$\frac{\sqrt{-4}}{2+\sqrt{-9}}$$

Answer:

$$\frac{2j}{2+3j} \times \frac{2-3j}{2-3j}$$

$$= \frac{4j-6j^2}{4+9}$$

$$= \frac{6+4j}{13}$$

For complete SLO's Preparation email to: **akber.khursheed@gamil.com**