Definition of Complex Numbers:

A complex number z is a number of the form $z = a + bi$

Where a and b are real numbers and i is the imaginary unit defined by $i = \sqrt{-1}$

a is called the real part of z and b is the imaginary part of z.

Note that the set **R** of all real numbers is a subset of the complex number **C** since any real number may be considered as having the imaginary part equal to zero..

Complex Conjugate:

The conjugate of a complex number **a + b*i*** is a complex number equal to **a - b*i***

Examples: Find the conjugate of the following complex numbers.

a) 2 - i , b) -3 + 4*i* , c) 5 , d) -5 *i*

Solution:

a) 2 + *i*, b) - 3 – 4 *i*, c) 5, d) 5*i*

Addition of Complex Numbers:

Addition of two complex numbers **a + b*i*** and **c + d*i*** is defined as follows.

(a + b *i*) + (c + d*i*) = (a + c) + (b + d)*i*

Real parts are added to real parts and imaginary parts are added to imaginary parts.

Example: Express in the form of a complex number **a + b *i*.**

Q.1 (2 + 3*i*) + (-4 + 5*i*) Q.2 (3*i*) + (-5 + *6i*) Q.3 (2) + (-2 + 9*i*)

Solution of Q.1:

- **(2 + 3i) + (-4 + 5i) = (2 - 4) + (3 + 5) i = - 2 + 8 i**

Solution of Q.2:

- **(3i) + (-5 + 6i) = (0 - 5) + (3 + 6) i = -5 + 9 i**

Solution of Q.3:

- **(2) + (-2 + 9i) = (2 - 2) + (9) i = 9i**

Alternatively Addition can be done by grouping like terms.

(2 + 3i) + (-4 + 5i) = 2 + 3 i - 4 + 5 i = -2 + 8 i

Subtraction of Complex Numbers:

The subtraction of two complex numbers **a + b *i*** and **c + d *i*** is defined as follows.

(a + b *i*) - (c + d *i*) = (a - b) + (b - d) *i*

Example: Express in the form of a complex number **a + b *i*.**

Q.1 (2 - 5i) - (- 4 - 5i) **Q.2 (-7i) - (- 5 - 6i)** **Q.3 (2) - (2 + 6i)**

Solution of Q.1:

- **(2 - 5i) - (- 4 - 5i) = (2 - (-4)) + (-5 - (-5)) i = 6**

Solution of Q.2:

- **(-7i) - (-5 - 6i) = (0 - (-5)) + (-7 - (-6)) i = 5 - i**

Solution of Q.3:

- **(2) - (2 + 6i) = (2 - 2) - 6 i = -6 i**

Subtraction can be done as follows:
(a + b *i*) - (c + d *i*) = (a + b *i*) + (- c - d *i*) and then make groups of like terms

Example:

(2 – 5 *i*) - (- 4 – 5 *i*) = 2 - 5 *i* + 4 + 5 *i* = 6

Multiply Complex Numbers:

The multiplication of two complex numbers **a + b *i*** and **c + d *i*** is defined as follows.

(a + b *i*) (c + d *i*) = (a c - b d) + (a d + b c) *i*

The multiplication can be carried out using properties similar to those of the real numbers and the added property $i^2 = -1$.

Example: Express the product in the form of a complex number **a + b *i*.**

Q.1 (3 + 2 *i*) . (3 – 3 *i*)

Solution:

(3 + 2 *i*)(3 – 3 *i*)

Using the **distributive law**, **(3 + 2 *i*) . (3 - 3 *i*)** can be written as

= (3 + 2 *i*) (3) + (3 + 2 *i*) (-3 *i*) = 9 + 6 *i* - 9 *i* - 6 i^2

using $i^2 = -1$

= 15 - 3 *i* Ans

Divide two Complex Numbers:

We use the multiplication property of complex number and it's conjugate to divide two complex numbers.

Example: Express in the form of a complex number **a + b *i.***

- $\dfrac{8+4i}{1-i}$

Multiply the numerator and denominator by the complex conjugate of the denominator

$$\frac{8+4i}{1-i} \times \frac{1+i}{1+i}$$

Multiply and make groups of like terms

$$= \frac{8-8i+4i+4i^2}{1+i-i-i^2} = \frac{4+12i}{2}$$

Work Sheet for Complex numbers:

- **Find the complex conjugate of the following complex numbers**

  a) 2 + 6 *i*

  b) -8 *i*

  c) 12

- **Write the following expressions in the form a + b *i***

  a) (2 - 8 *i*) + (-6 *i*)

  b) -8 *i* + (3 - 9 *i*)

c) 6 - (3 – *i*)

d) (2 - 3 *i*)(7 – *i*)

e) (2 + 2 *i*) / (2 – 2 *i*)

- **<u>Find the complex conjugate.</u>**

  a) 2 - 6 *i*

  b) 8 *i*

  c) 12

<u>Write the following expressions in the form a + b *i*</u>

a) (2 - 8 *i*) + (- 6 *i*) = 2 - 14 *i*

b) -8 *i* + (3 - 9 *i*) = 3 - 17 *i*

c) 6 - (3 – *i*) = 3 + *i*

d) (2 - 3 *i*) (7 – *i*) = 11 - 23 *i*

e) (2 + 2 *i*) / (2 - 2 *i*) = *i*