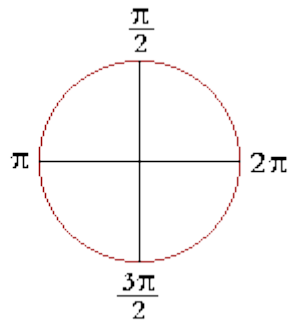


IN THE RADIAN SYSTEM

The angular measurement, the measure of one revolution is 2π .



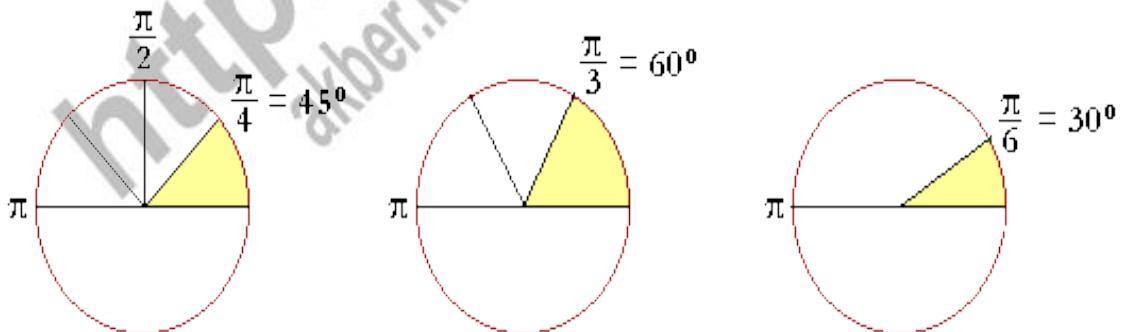
Half a circle, then, is π . And, most important, each right angle is half of π : $\frac{\pi}{2}$.

Three right angles will be $3 \cdot \frac{\pi}{2} = \frac{3\pi}{2}$.

Five right angles will be $\frac{5\pi}{2}$. And so on.

Radians into degrees

The following radian measures come up frequently, and the student should know their degree equivalents:

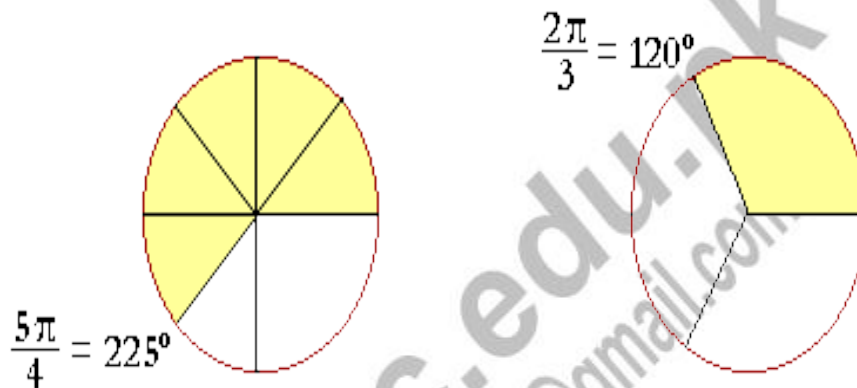


$\frac{\pi}{4}$ is half of $\frac{\pi}{2}$, a right angle, and so it is equal to 45° .

Equivalently, $\frac{\pi}{4}$ is of one quarter, Half half of π , which is 180° .

$\frac{\pi}{3}$ is a third of π , and so is equal to $180^\circ \div 3 = 60^\circ$.

$\frac{\pi}{6}$ is a sixth of π , and so is equal to $180^\circ \div 6 = 30^\circ$.



$$\frac{5\pi}{4} = 5 \cdot \frac{\pi}{4} = 5 \cdot 45^\circ = 225^\circ.$$

$\frac{2\pi}{3}$ is a third of 2π . A third of a revolution = $360^\circ \div 3 = 120^\circ$.

Problem 1: Convert each of these radian measures into degrees.

- a) π 180° b) $\frac{\pi}{2}$ 90° c) $\frac{\pi}{3}$ 60° d) $\frac{\pi}{6}$ 30° e) $\frac{\pi}{4}$ 45°

Problem 2: Convert each of these radian measures into degrees.

a) $\frac{\pi}{8}$ $22\frac{1}{2}^\circ$. $\frac{\pi}{8}$ is half of $\frac{\pi}{4}$.

b) $\frac{2\pi}{5}$ 72° . $\frac{2\pi}{5}$ is a fifth of 2π , which is a fifth of a 360° .

c) $\frac{7\pi}{4} = 7 \cdot \frac{\pi}{4} = 7 \cdot 45^\circ = 315^\circ$

d) $\frac{9\pi}{2} = 9 \cdot \frac{\pi}{2} = 9 \cdot 90^\circ = 810^\circ$

e) $\frac{4\pi}{3} = 4 \cdot \frac{\pi}{3} = 4 \cdot 60^\circ = 240^\circ$

f) $\frac{5\pi}{6} = 5 \cdot \frac{\pi}{6} = 5 \cdot 30^\circ = 150^\circ$

g) $\frac{7\pi}{9} = \frac{7}{9} \cdot \pi = \frac{7}{9} \cdot 180^\circ = \frac{7}{9} \cdot 180^\circ = 140^\circ$

Degrees into radians:

$$360^\circ = 2\pi. \quad 180^\circ = \pi.$$

Example 1: Convert 120° into radians.

Solution: Since

$$60^\circ = \frac{\pi}{3}, \text{ then}$$

$$120^\circ = 2 \cdot 60^\circ = 2 \cdot \frac{\pi}{3} = \frac{2\pi}{3}.$$

Or, since 120° is a third of 360° , which is 2π , then

$$120^\circ = \frac{2\pi}{3}.$$

Example 2: $225^\circ = 180^\circ + 45^\circ = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$

In general, proportionally,

$$\frac{\text{Radians}}{\pi} = \frac{\text{Degrees}}{180^\circ}$$

$$\text{Radians} = \frac{\text{Degrees}}{180} \cdot \pi$$

Example 3: Change 140° to radians.

Solution. $\frac{140}{180} \cdot \pi = \frac{7}{9} \cdot \pi = \frac{7\pi}{9}$,

Problem 6: Change each of the following into radians.

i) $210^\circ = 7 \cdot 30^\circ = 7 \cdot \frac{\pi}{6} = \frac{7\pi}{6}$

j) $300^\circ = 5 \cdot 60^\circ = 5 \cdot \frac{\pi}{3} = \frac{5\pi}{3}$

k) $135^\circ = 90^\circ + 45^\circ = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

l) $72^\circ = \frac{72}{180} \cdot \pi = \frac{2}{5} \cdot \pi = \frac{2\pi}{5}$