

Complex numbers:

Complex numbers have the form $a + i b$ where a and b are real numbers.

The i in the complex number refers to $i = \sqrt{-1}$ which allows for solving equations which have imaginary roots - that is the discriminate $(b^2 - 4ac)$ is less than zero. It's easy to see that squaring both sides of the i equation above yields

$$i^2 = -1.$$

This is the basis for all the imaginary complex numbers.

Basic Operations of Complex Numbers:

Now, these numbers can be added, subtracted, multiplied and divided in exactly the same manner as all the real numbers. Indeed, the rules are the same!

Example:

Suppose two complex numbers, say $5 - 2i$ and $3 + 4i$.

To add: $(5 - 2i) + (3 + 4i) = 8 + 2i$ merely combine component parts. $5 + 3$
and $-2i + 4i$

Subtraction: $(5 - 2i) - (3 + 4i) = 5 - 2i - 3 - 4i = 2 - 6i$

Multiplication :

$$(5 - 2i)(3 + 4i) = 15 - 6i + 20i - 8i^2 \text{ but } i^2 = -1 \text{ so } -8i^2$$

$$\text{becomes } +8 \rightarrow 23 + 14i$$

this is equal to -1

$$= 23 + 14i$$

Divide: Simplify the following:

$$(5 - 2i) \div (3 + 4i) = \frac{5 - 2i}{3 + 4i}$$

Multiply by the conjugate of the denominator (in the form of 1)

$$\frac{5 - 2i}{3 + 4i} \cdot \frac{3 - 4i}{3 - 4i} = \frac{(5 - 2i)(3 - 4i)}{9 - 16i^2} = \frac{15 - 6i - 20i + 8i^2}{9 + 16}$$

$$= \frac{7 - 26i}{25} = \frac{7}{25} - \frac{26}{25}i$$

Simplify, combine like terms and write in standard form.

1. $4 - 3i + 9i^2 - (\sqrt{16} + \sqrt{-16})$

2. $(4 - i) + (1 + 3i)$

3. $(4 - i)(1 + 3i)$

4. $(5 + 7i) - (1 - i)$

5. $6 + (4 - 3i)$

6. $(3 + 4i)(2 - 5i)$

7. $(2i) - (3 + 7i)$

8. $(7 + i)(4 - 5i)$

9. $(7 + i) \div (4 - 5i)$

10. $(4 - 5i) - (7 + i)$

11. $(6 + 7i) + \sqrt{9i} - (4 + 3i) - 2i + 1$

12. $\frac{4 - 5i}{3 + i}$

13. $(3 - 11i) - (3 + 11i)$

14. $(2 + 5i)(2 - 5i)$

15. $(3 - 4i)^2$