

Quadrantal Angle:

Definition of Quadrantal Angle:

- An angle in standard position with terminal side lying on x -axis or y -axis is called as Quadrantal Angle.

More about Quadrantal Angle

- The quadrantal angles are 0° , 90° , 180° , 270° , 360° , 450° , etc. and -90° , -180° , -270° , -360° , etc.
- Some of the trigonometric functions are undefined for some quadrantal angles.

Examples of Quadrantal Angle:

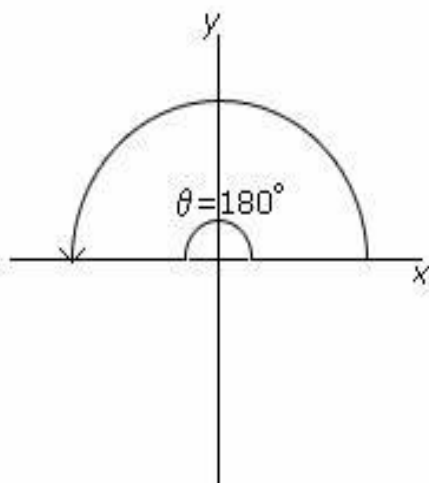


Figure 1

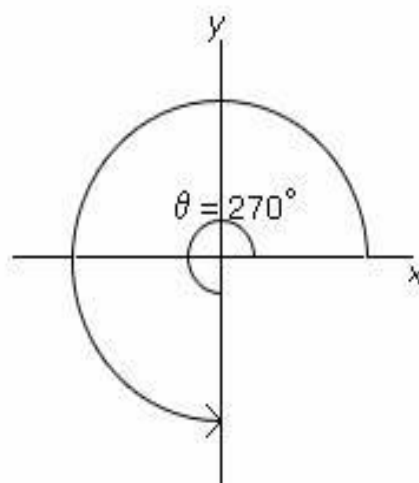


Figure 2

- Figure 1 shows the second quadrant angle of 180° with the terminal side on the x -axis and Figure 2 shows the third quadrant angle of 270° with the terminal side on the y -axis.

Solved Example on Quadrantal Angle:

Identify the quadrantal angle.

Choices:

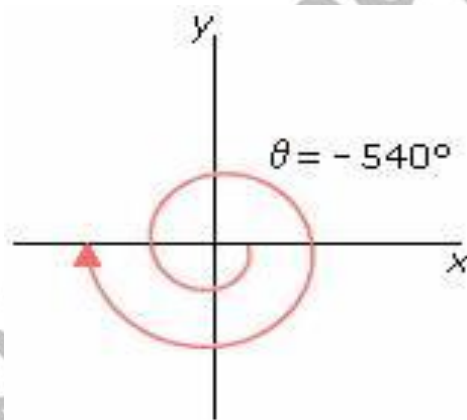
- A. 210°
- B. -180°
- C. 345°
- D. -540°

Correct Answer: D

Solution:

Step 1: A quadrantal angle is an angle in standard position whose terminal side lies on x -axis or y -axis.

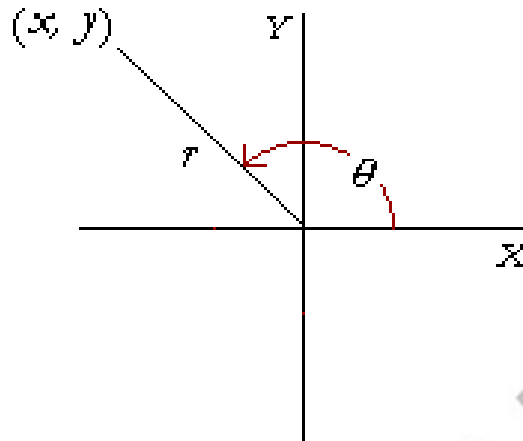
Step 2: Here, -540° is the third quadrant angle with the terminal side on x -axis. [In clockwise direction the measure is negative.]



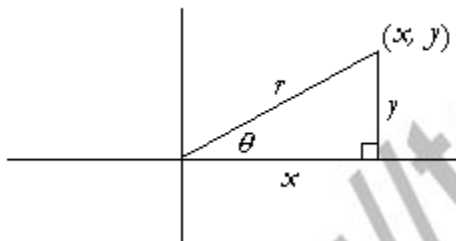
Step 3: So, -540° is the quadrantal angle.

- Figure 1 shows the second quadrant angle of 180° with the terminal side on the x -axis and Figure 2 shows the third quadrant angle of 270° with the terminal side on the y -axis.
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ANALYTIC TRIGONOMETRY on the x - y plane.



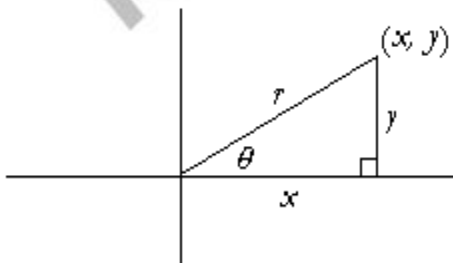
- Let a straight line of length r sweep out an angle θ and let the Co-ordinates of its extremity be (x, y) . Now we shall define the six trigonometric functions of θ
- We will take our cue from the first quadrant. In that quadrant,



- a radius r will terminate at a point (x, y) . Those co-ordinates define a right triangle. The six trigonometric functions are as follow.

$$r = \sqrt{x^2 + y^2}$$

- This is the analytic definition for an angle terminating in *any* quadrant. It is in terms of the co-ordinates (x, y) of the endpoint of a distance r from the origin.
- But before we give an example, consider this question:



- **Example 1.** A straight line inserted at the origin terminates at the point (3, 2) as it sweeps out an angle θ in standard position. Evaluate all six functions of θ .

- **Answer.** $x = 3$, $y = 2$. Therefore, according to the [definitions](#):

$$r = \sqrt{x^2 + y^2} = \sqrt{3^2 + 2^2} = \sqrt{13}$$

$$\sin \theta = \frac{y}{r} = \frac{2}{\sqrt{13}} \quad \csc \theta = \frac{r}{y} = \frac{\sqrt{13}}{2}$$

$$\cos \theta = \frac{x}{r} = \frac{3}{\sqrt{13}} \quad \sec \theta = \frac{r}{x} = \frac{\sqrt{13}}{3}$$

$$\tan \theta = \frac{y}{x} = \frac{2}{3} \quad \cot \theta = \frac{x}{y} = \frac{3}{2}$$

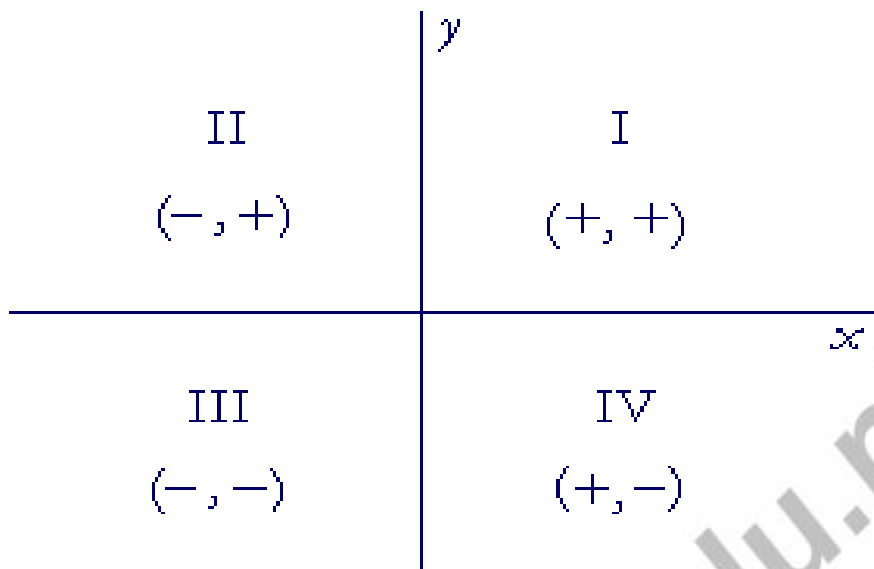
- **Problem 1.** A straight line from the origin sweeps out an angle θ , and it terminates at the point (3, -4). Evaluate the six functions of θ .

$$\sin \theta = -\frac{4}{5} \quad \csc \theta = -\frac{5}{4}$$

$$\cos \theta = \frac{3}{5} \quad \sec \theta = \frac{5}{3}$$

$$\tan \theta = -\frac{4}{3} \quad \cot \theta = -\frac{3}{4}$$

- **Problem 2.** The signs in each quadrant.



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• Quadrantal angles:

- A quadrantal angle is an angle that terminates *on* the x - or y -axis.

- **Problem 3.**

- a) What are the quadrantal angles in degrees?
- $0^\circ, 90^\circ, 180^\circ, 270^\circ$; and angles coterminal with them.

- b) What are the quadrantal angles in radians?

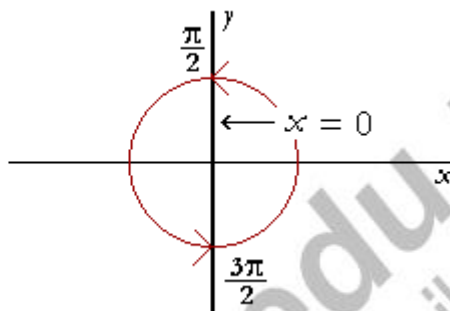
$0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$; and angles co-terminal with them.

- c) When an angle terminates on the x -axis, what is the value of the y -coördinate?
- 0. On the x -axis, $y = 0$.
- d) When an angle terminates on the y -axis, what is the value of the x -coördinate?
- 0. On the y -axis, $x = 0$.
- Now, it is a fact of arithmetic that there is no *number* with denominator 0.

- Therefore, wherever a trigonometric function has a denominator x or y -- equal to 0, the function will not exist at that quadrantal angle.
- For example,

$$\tan \theta = \frac{y}{x}$$

- Wherever $x = 0$, $\tan \theta$ will not exist. Why $x = 0$? When the angle terminates on the y -axis.



- $\tan \theta$ will not exist at $\theta = \frac{\pi}{2}$ and $\theta = \frac{3\pi}{2}$ or $-\frac{\pi}{2}$, which is co-terminal.
- Those values of θ will be singularities of $\tan \theta$.
- **Problem 4.** For which quadrantal angles do the following functions not exist?

a) $\cot \theta$ $\cot \theta = \frac{x}{y}$. Therefore, when ever $y = 0$ -- that is, on

- b) $\sec \theta$

$\sec \theta = \frac{r}{x}$. Therefore, $\sec \theta$ will not exist where $x = 0$,

which is on the y -axis. $\sec \theta$ will not exist at $\theta = \frac{\pi}{2}$

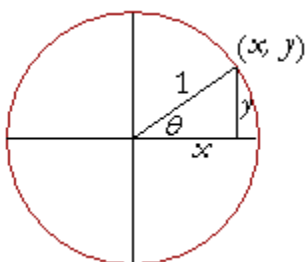
and $\theta = \frac{3\pi}{2}$.

- c) $\sin \theta$

$\sin \theta = \frac{y}{r}$. But r is never 0. There is no angle for which

• The unit circle:

- Therefore we may choose any radius we please, and the simplest is a circle of radius 1, the **unit circle**.



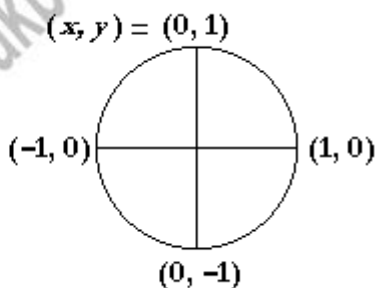
- On the unit circle the functions take a particularly simple form. For example,

$$\sin \theta = \frac{y}{1} = y.$$

$$\cos \theta = \frac{x}{1} = x.$$

- The value of $\sin \theta$ is the y -coordinate of the unit radius!
- The value of $\cos \theta$ is the x -coordinate!

- The unit circle illustrates the following:



- If a function exists at a Quadrant angle, it could have only the values 0, 1, or -1 .

- Consider $\sin \theta$ at each quadrantal angle. We just saw that the value of $\sin \theta$ is the y -coordinate:

- $\sin \theta = y$.

- Therefore at each quadrantal angle, the value of $\sin \theta$ -- of y -- is either 0, 1, or -1 .

At $\theta = 0$, $\sin \theta = 0$.

At $\theta = \frac{\pi}{2}$, $\sin \theta = 1$.

At $\theta = \pi$, $\sin \theta = 0$.

At $\theta = \frac{3\pi}{2}$, $\sin \theta = -1$.

- Problem 5.** Evaluate the following.

- a) $\cos 0^\circ$

- b) $\cos 90^\circ = 0$ c) $\cos 180^\circ = -1$ d) $\cos 270^\circ = 0$

- e) $\tan 0^\circ$

- f) $\tan 90^\circ = 1/0$ does not exist. g) $\tan 180^\circ = 0$

- h) $\tan 270^\circ$ does not exist.

- Problem 6.** Evaluate the following -- if it exists.

a) $\cos \frac{\pi}{2}$

b) $\sin \frac{\pi}{2} = 1$ c) $\sin \pi = 0$ d) $\cos \pi = -1$

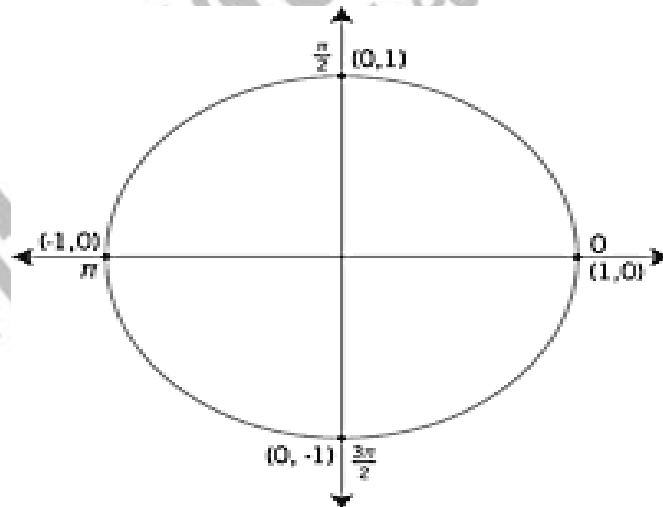
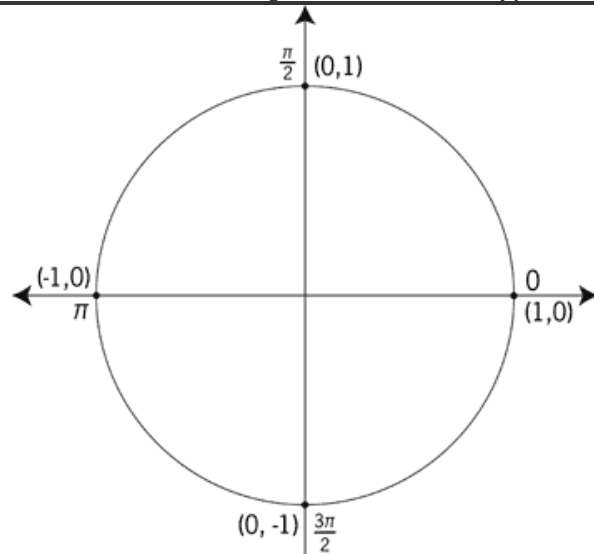
e) $\cot 0 = x/y$. **Does not exist.** f) $\cot \frac{\pi}{2} = 0$

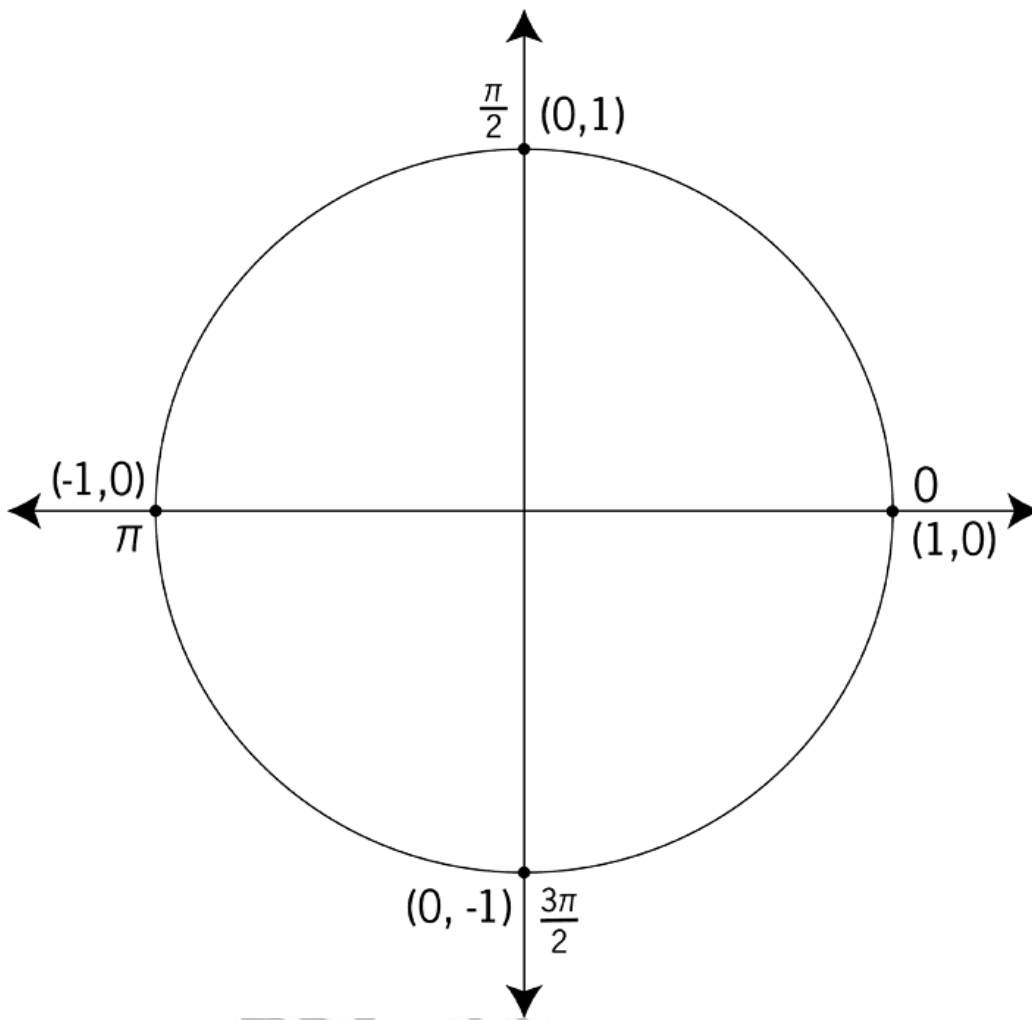
g) $\tan \frac{\pi}{2}$ **Does not exist.** h) $\sec 0 = 1/x = 1$ i) $\csc \left(-\frac{\pi}{2}\right) = -1$

j) $\sin 2\pi = 0$ k) $\sin 3\pi = 0$ l) $\sin 4\pi = 0$ m) $\sin (-\pi) = 0$

n) $\cos 2\pi = 1$ o) $\cos 3\pi = -1$ p) $\cos 4\pi = 1$ q) $\cos 5\pi = -1$

- Unit Circle Labeled With Quadrantal Angles And Values:





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